$$U(s) = C(s)\, E(s)$$

$u(t) \qquad\qquad e(t)$

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1 - e^{-\tau_D s})}\, E(s)$$

$$U(s)\, K(\tau_c s + 1 - e^{-\tau_D s}) = (\tau s + 1)\; E(s)$$

$$\frac{K\tau_c s}{\tau s + 1} U(s) = E(s) - \frac{K}{\tau s + 1} U(s) + \frac{K e^{-\tau_D s}}{\tau s + 1} U(s)$$

$$\frac{K\tau_c s}{\tau s+1}\, U(s) = E(s) - (G_M^* - G_M)U(s),$$

where G_M^* is the model of the process without the delay term

$$U(s) = \frac{\tau s + 1}{K\tau_c s}\left(E(s) - (G_M^* - G_M)U(s)\right)$$

- $\frac{\tau s+1}{K\tau_c s}$ represents a PI controller of the form $\frac{\tau}{K\tau_c}\left(1 + \frac{1}{\tau s}\right)$ where the gain of the controller is $\frac{\tau}{K\tau_c}$ and the integral constant is τ
- The error term $E(s)$ is corrected as $\left(E(s) - (G_M^* - G_M)U(s)\right)$ using the model information

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$U(s) = C(s)\, E(s)$

$u(t)$ $\quad$ $e(t)$

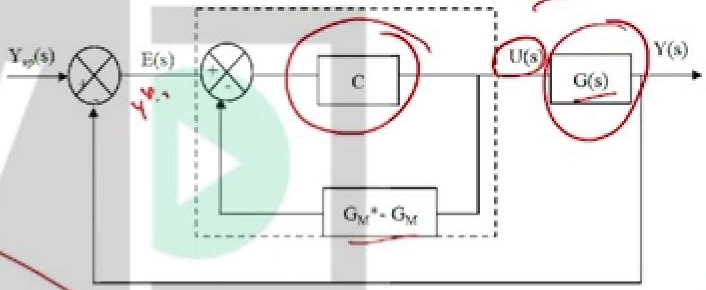
$$\frac{K\tau_c s}{\tau s+1} U(s) + \frac{K}{\tau s+1} U(s) - \frac{K e^{-\tau_D s}}{\tau s+1} U(s)$$

$U(s) = \boxed{PI}\; E(s)$

Controller design for time delay systems – analysis of model information

Model information is used in the controller calculations

$$Y^{des} = \frac{e^{-\tau_D s}}{\tau_C s+1}$$



$$\boxed{U(s) = \frac{\tau s+1}{K\tau_c s}\left(E(s) - (G_M^* - G_M)U(s)\right)}$$

$$U(s) = \frac{\tau s+1}{K\tau_c s}\left(Y^{sp} - Y(s) - (G_M^* - G_M)U(s)\right)$$

$G(s)U(s)$

If the model is perfect, then $G(s)=G_M(s)$ and $Y(s)=G(s)U(s)= G_M(s)\,U(s)$.

$$Y(s) = G(s)U(s)$$
$$= G_M(s)U(s)$$

$$U(s) = \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - Y(s) - (G_M^* - G_M)U(s)\right)$$

$$= \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - \cancel{G_M}(s)U(s) - (G_M^* - \cancel{G_M})U(s)\right)$$

$$= \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - G_M^*(s)U(s)\right)$$

Controller design for time delay systems – analysis of model information

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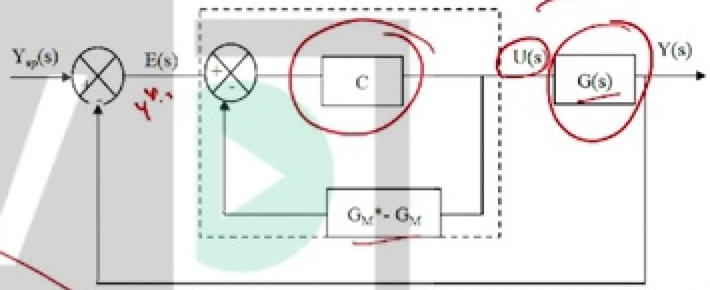

$$\boxed{U(s) = \frac{\tau s+1}{K\tau_c s}\left(E(s) - (G_M^* - G_M)U(s)\right)}$$

$$U(s) = \frac{\tau s+1}{K\tau_c s}\left(Y^{sp} - \underbrace{Y(s)}_{G(s)U(s)} - (G_M^* - G_M)U(s)\right)$$

$$Y(s) = G(s)U(s) = G_m(s)U(s)$$

If the model is perfect, then $G(s)=G_M(s)$ and $Y(s)=G(s)U(s)= G_M(s)\, U(s)$.

$$U(s) = \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - Y(s) - (G_M^* - G_M)U(s)\right)$$

$$= \quad = \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - \cancel{G_M}(s)\mathrm{U}(s) - (G_M^* - \cancel{G_M})U(s)\right)$$

$$= \frac{\tau s+1}{K\tau_c s}\left(Y_{sp}(s) - G_M^*(s)U(s)\right)$$

Transfer function inversion without approximation – another interpretation

Consider controller TF between the input and the error

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1) - e^{-\tau_D s}} E(s)$$

$$\frac{K(\tau_c s + 1)}{\tau s + 1} U(s) = E(s) + \frac{K e^{-\tau_D s}}{\tau s + 1} U(s)$$

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1)} E(s) + \frac{e^{-\tau_D s}}{(\tau_c s + 1)} U(s)$$

Let $e^{-\tau_D s}$ be denoted as $U^d(s)$

$$U(s) = \frac{\tau s + 1}{K(\tau_c s + 1)} E(s) + \frac{1}{(\tau_c s + 1)} U^d(s)$$

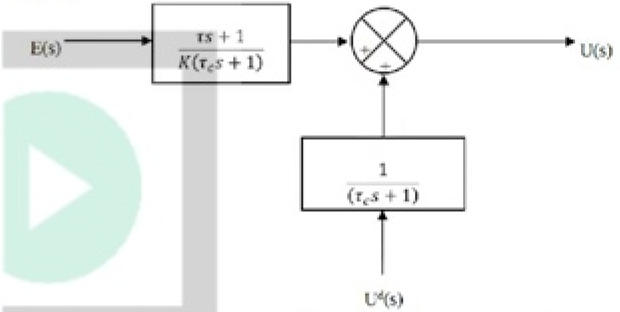

Fig: Inverting TF with time delay without approximation

$$\boxed{e^{-\tau_D s} U(s)} = U^d(s)$$

$u(t)$

$u(t - \tau_D)$